微積分的應用：RC電路
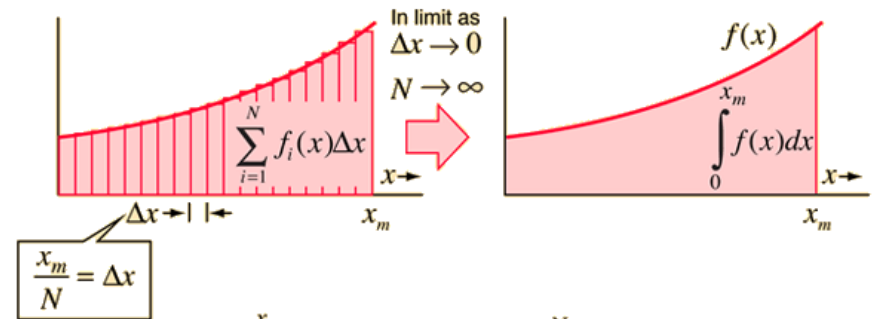
「微分」在意物理量的變化率
「積分」關心物理量的累積

Sum becomes Integral

$$Area = \int_0^{x_m} f(x)dx = \lim_{\Delta x \to 0} \sum_{i=1}^{N} f_i(x)\Delta x$$
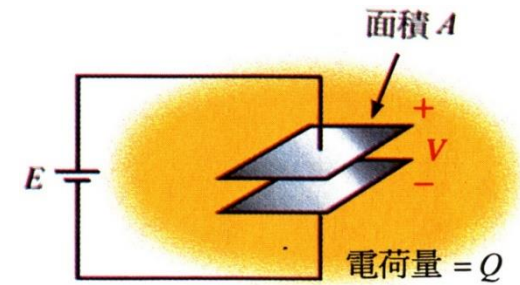
學習目標

- 瞭解RC(電阻+電容)電路在生活中的應用
- 瞭解RC電路的工作原理
- 以方程式描述RC電路的充放電行為
  - 以微積分式表達電流與電壓之間的關係
  - 推導電路的特性
  - 延伸方程式的內涵，反思工程實務上的課題
- RC電路的暫態特性於其他工程的應用
- 以軟體工具探討RC電路的脈波響應
  - 程式撰寫
  - 不同的輸入形式與輸出的電壓之間的關係

知其然，也知其所以然

Review: RC電路

- RC電路
  - 電阻(<u>r</u>esistor)+電容(<u>c</u>apacitor)的基本電路
  - 運用電容充電、蓄電、放電的物理特性
  - 常用於定時、整流、震盪有關的應用
    - (照相機)閃光燈
    - 定時關燈裝置
    - 濾波器 *http://bit.ly/1PW29X6*
    - 電蚊拍
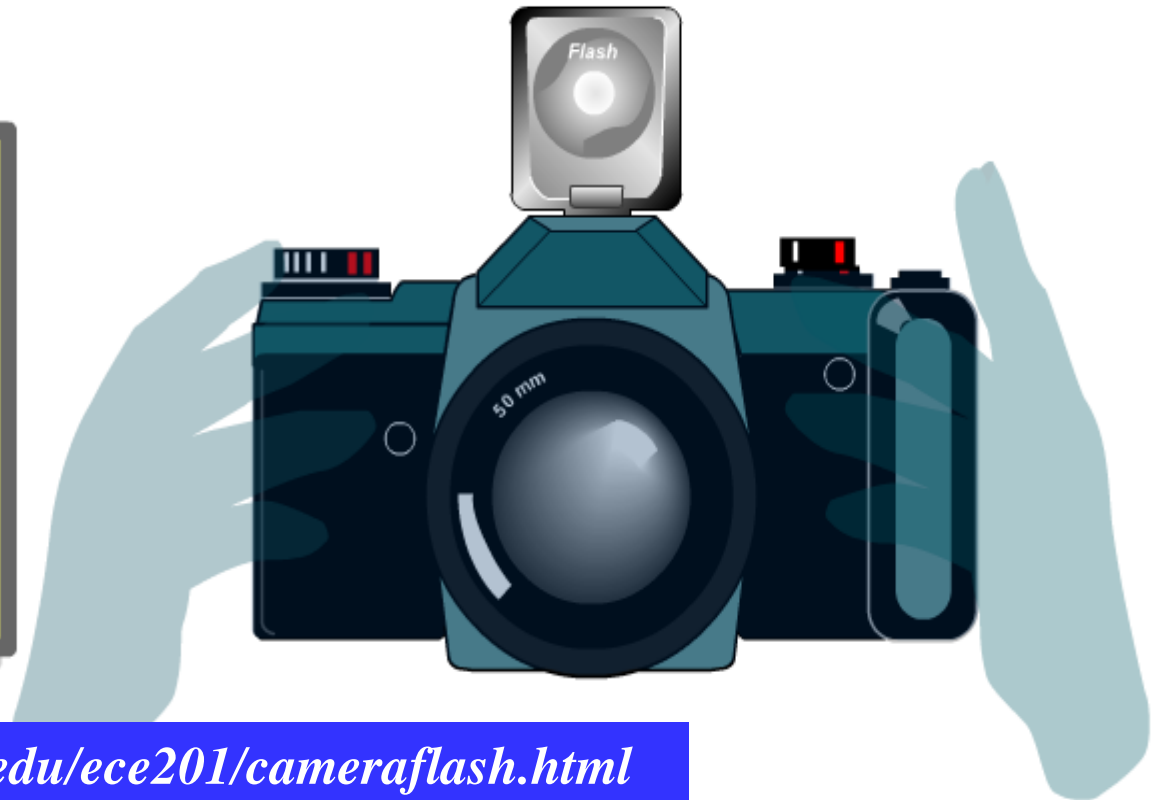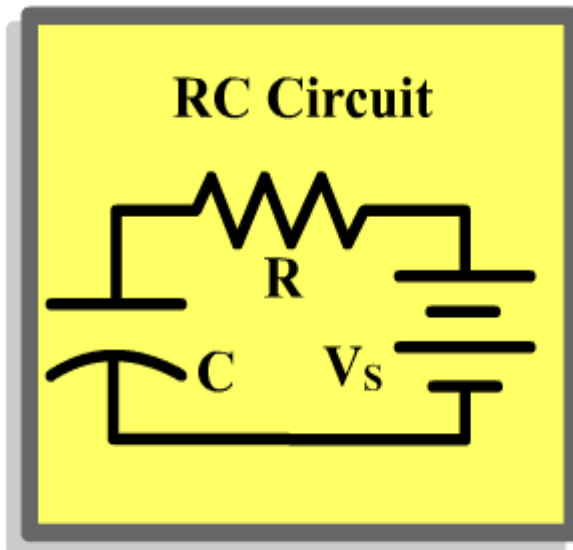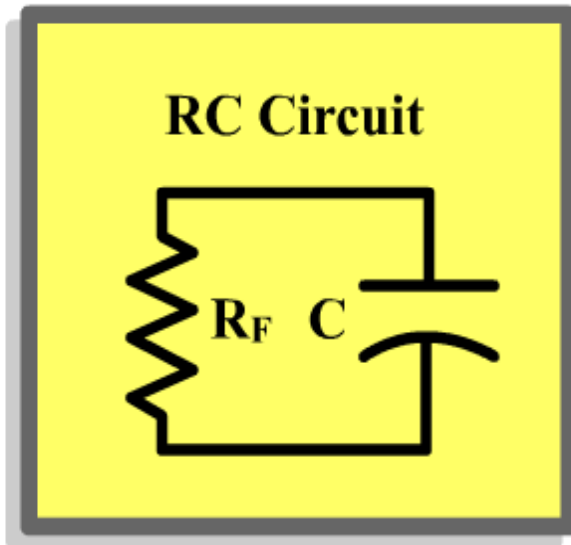   - …

Camera Flash (RC Circuits)

When the camera is switched on, a battery slowly charges a capacitor to its full potential via an RC circuit.

RC Circuit

http://holbert.faculty.asu.edu/ece201/cameraflash.html
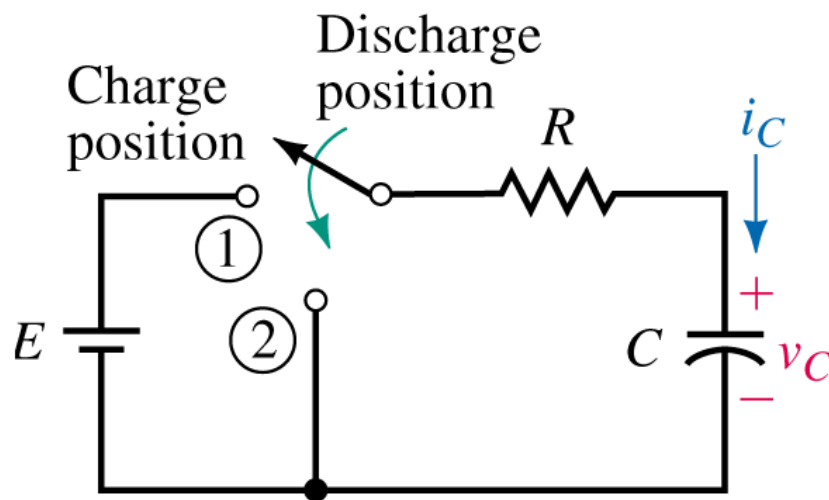
Camera Flash (RC Circuits)

The photographer then presses the shutter button, quickly discharging the capacitor through the flash (a resistor). The current from the capacitor is responsible for the bright light that is emitted.

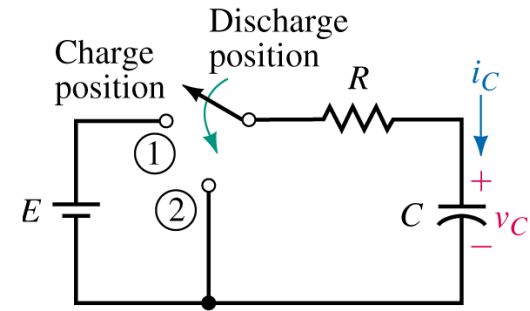**RC Circuit**

R_F C

Review: 電容的充電放電

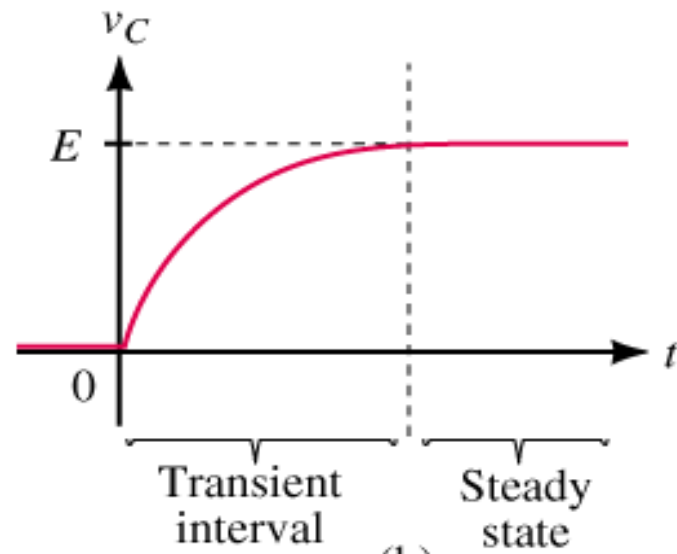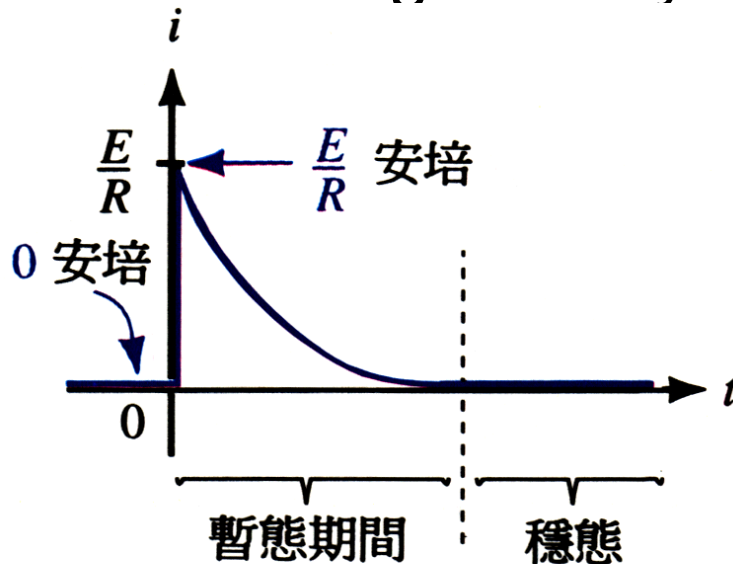- RC circuit
  - 有哪些其他應用？

  - Capacitor charging (充電) and discharging (放電)
  - Transient (瞬間的) voltages and currents result when the circuit is switched
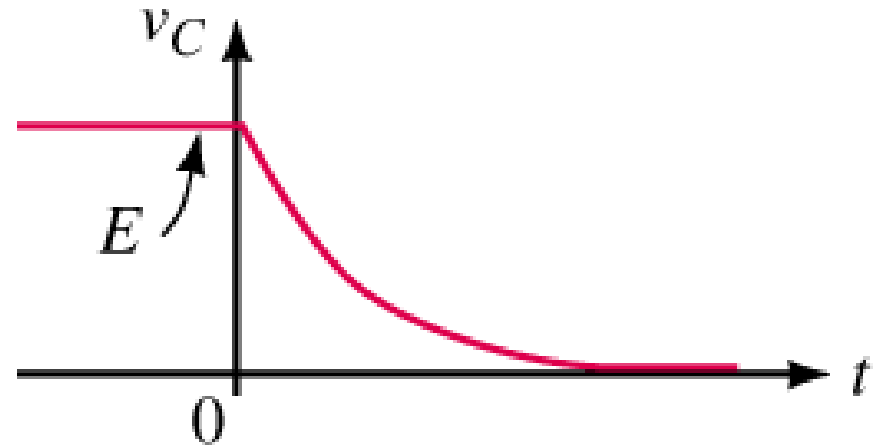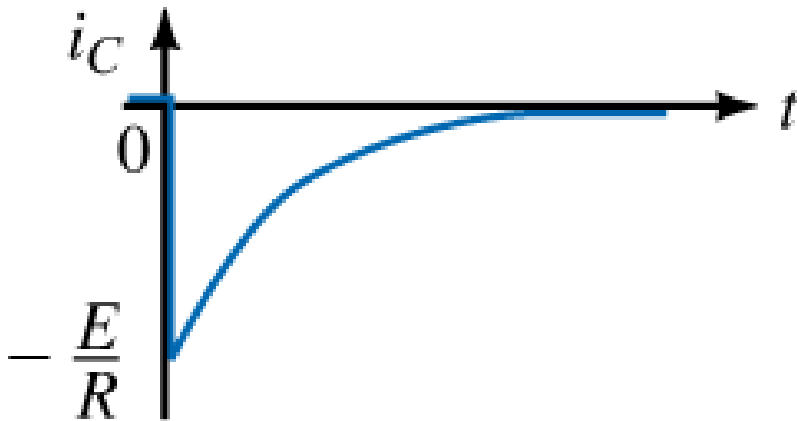
Review: 電容的充電

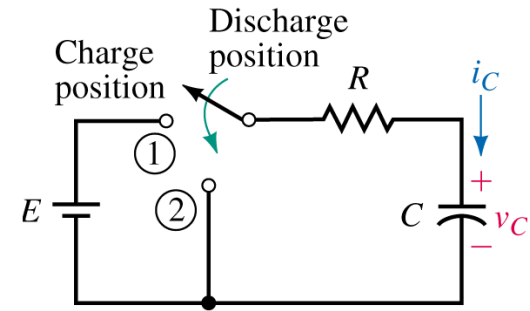

- For an uncharged capacitor, at the instant the switch is closed, the current jumps to E/R, then decays to zero

  - At the instant of switching, the circuit looks like a short circuit

    v_C在開關切換瞬間仍保持0，但此時電子開始移動，形成電流 ∵電位差=0，此時猶如電容二接腳暢通，形同短路

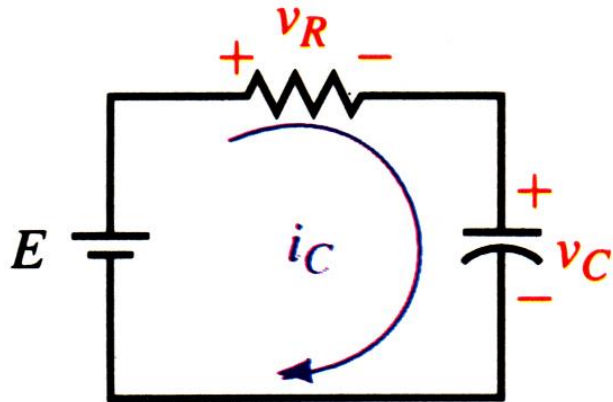 - The voltage across the capacitor begins at zero and gradually climbs to E volts

Review: 電容的放電

- Assume the capacitor has E volts across when it begins to discharge

  - The current jumps to $-E/R$

  - Both voltage and current will decay to zero

Capacitor Charging Equations

- Voltages and currents in a charging circuit do not change instantaneously
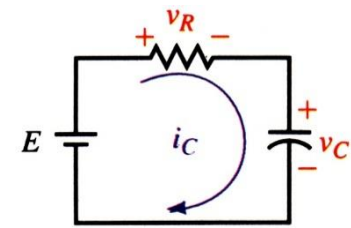 - These changes over time exhibit an exponential form

By KVL, $v_R + v_C = E$

$$\because v_R = R\boxed{i_C} = R(\boxed{C\frac{dv_C}{dt}})$$

$$\therefore RC\frac{dv_C}{dt} + v_C = E$$

Integration leads to $\boxed{v_C = E\left(1 - e^{-t/RC}\right)}$

WHY?

Capacitor Charging Equations

$$RC\frac{dv_C}{dt} + v_C = E$$

$$-\int_{V_0}^{v} \frac{1}{E-v_C} d(E-v_C) = \int_{0}^{t} \frac{1}{RC} dt$$

$$RC\frac{dv_C}{dt} = E - v_C$$

$$-\ln(E-v_C)\Big|_{V_0}^{v} = \frac{t}{RC}$$

$$\frac{1}{E-v_C} dv_C = \frac{1}{RC} dt$$

$$-(\ln(E-v) - \ln(E-V_0)) = \frac{t}{RC}$$

$$-\frac{1}{E-v_C} d(E-v_C) = \frac{1}{RC} dt$$

$$\ln\frac{E-v}{E-V_0} = -\frac{t}{RC}$$

"="左右二邊進行積分運算   "="左右二邊取指數運算

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Capacitor Charging Equations

$$\ln\frac{E-v}{E-V_0} = -\frac{t}{RC}$$

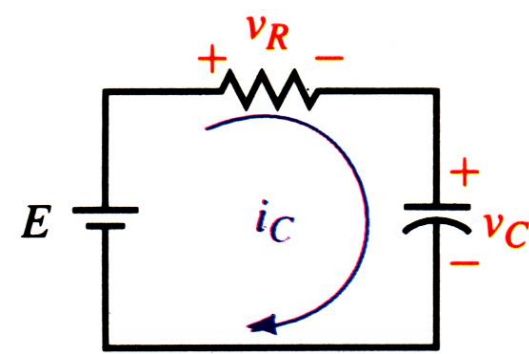"="左右二邊取指數運算得： $e^{\ln\frac{E-v}{E-V_0}} = \frac{E-v}{E-V_0} = e^{-\frac{t}{RC}}$

$$E-v = (E-V_0)e^{-\frac{t}{RC}}$$

$$\therefore v = E(1-e^{-\frac{t}{RC}}) + V_0 e^{-\frac{t}{RC}}$$

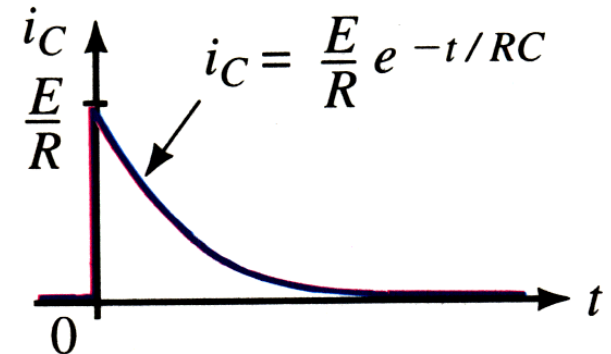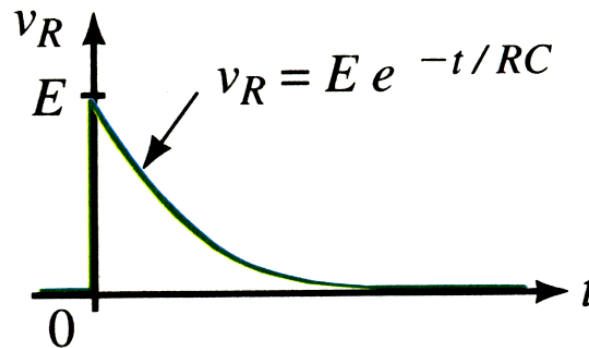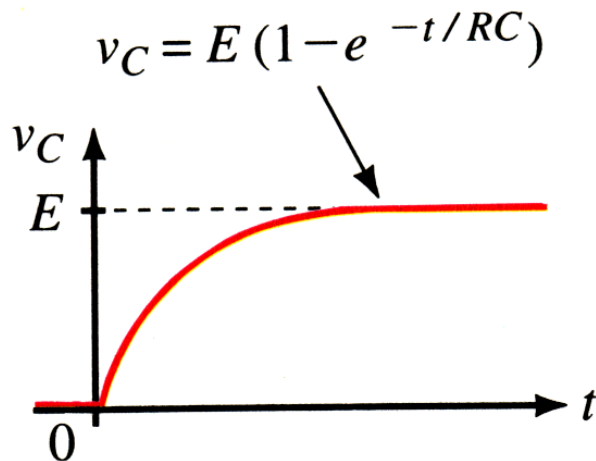$$= \boxed{E + (V_0 - E)e^{-\frac{t}{RC}}}$$

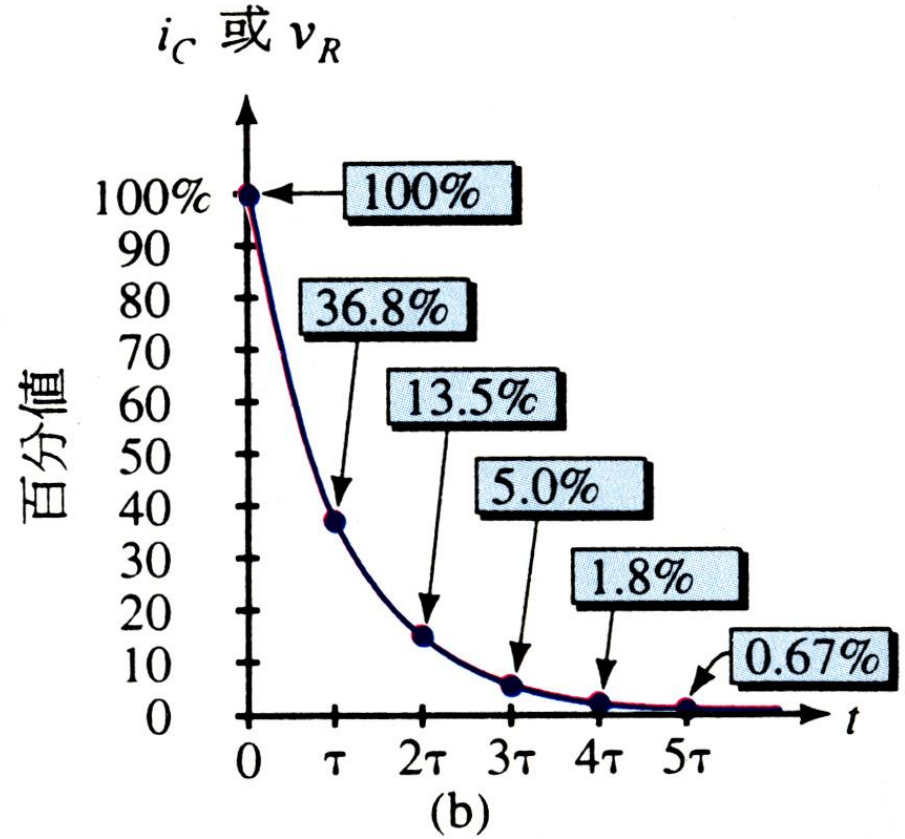11

Capacitor Charging Equat

- Voltage across the resistor

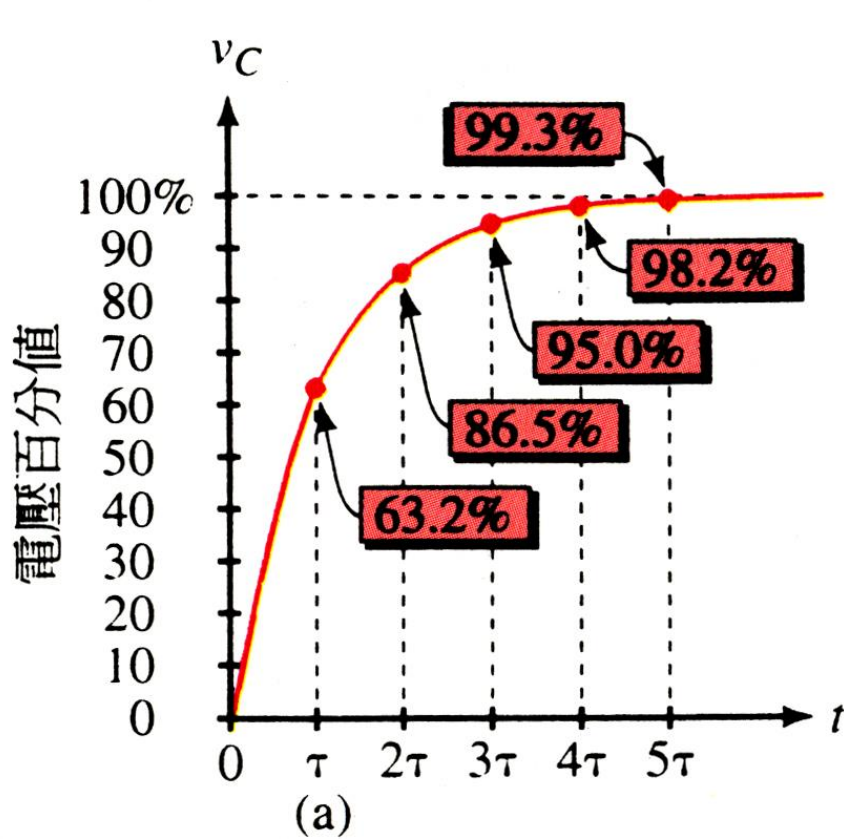$$v_R = E - v_C$$

$$= E - E\left(1 - e^{-t/RC}\right) = Ee^{-t/RC}$$

$$\Rightarrow \frac{v_R}{R} = i_R = \boxed{i_C = \frac{E}{R}e^{-t/RC}} \quad \cdots \quad 電路中的電流值$$

$$v_C = E(1 - e^{-t/RC})$$

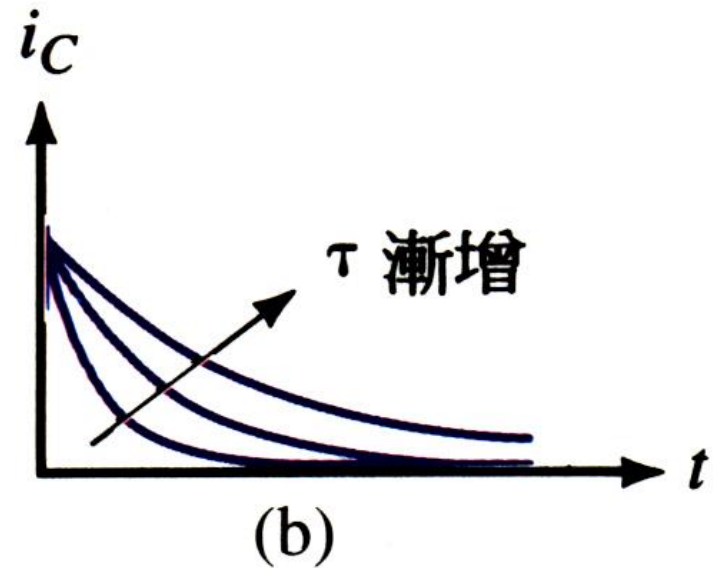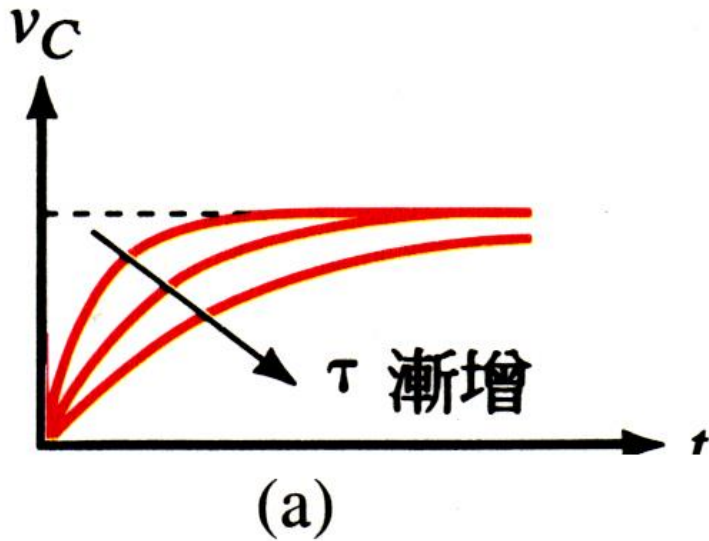$$v_R = Ee^{-t/RC}$$

$$i_C = \frac{E}{R}e^{-t/RC}$$

Duration of a Transient

- 充電過程中的電壓及電流曲線 (時間常數$\tau = RC$)

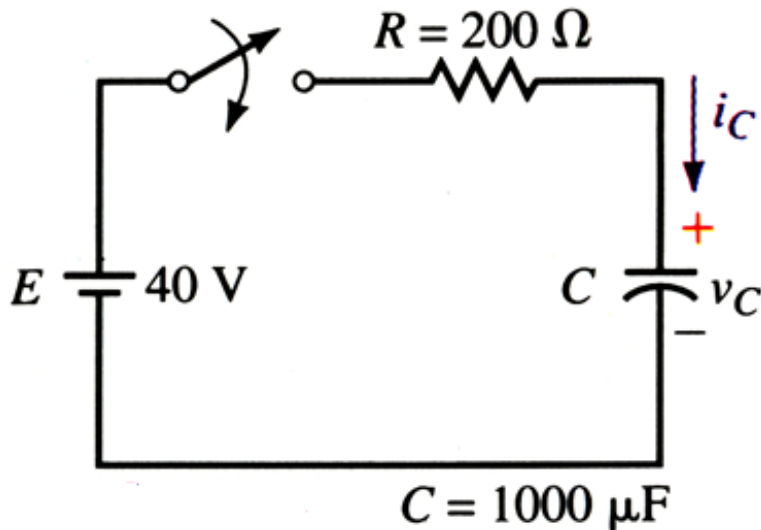# Duration of a Transient

- 時間常數τ越大，需更多時間充電

(a)

(b)

$$v_C = E\left(1 - e^{-t/RC}\right)$$

$$i_R = i_C = \frac{E}{R}e^{-t/RC}$$

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Capacitor with an Initial Voltage

- If the capacitor already has a voltage on it, this voltage is denoted as V_0
- Voltage and current in a circuit will be affected by the initial voltage
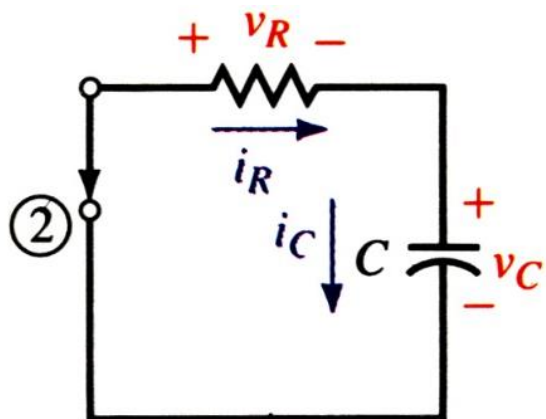
$$v_C = E + (V_0 - E)e^{-t/\tau}$$

$$i_C = \frac{E - V_0}{R}e^{-t/\tau}$$

(☆)

$R = 200\ \Omega$

E — 40 V

i_C

C — v_C

$C = 1000\ \mu F$

Capacitor Discharging Equations

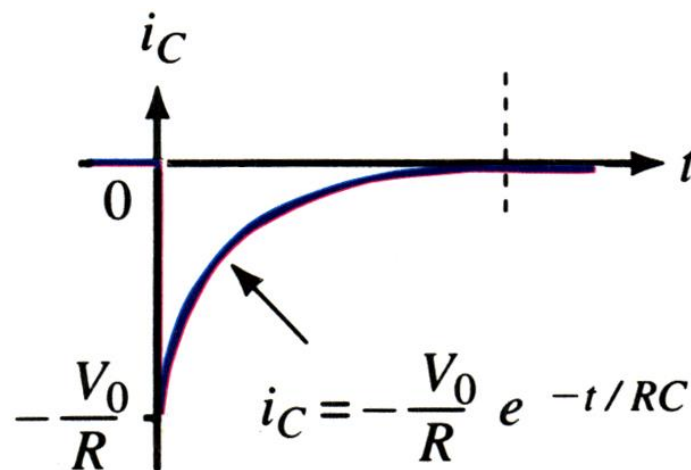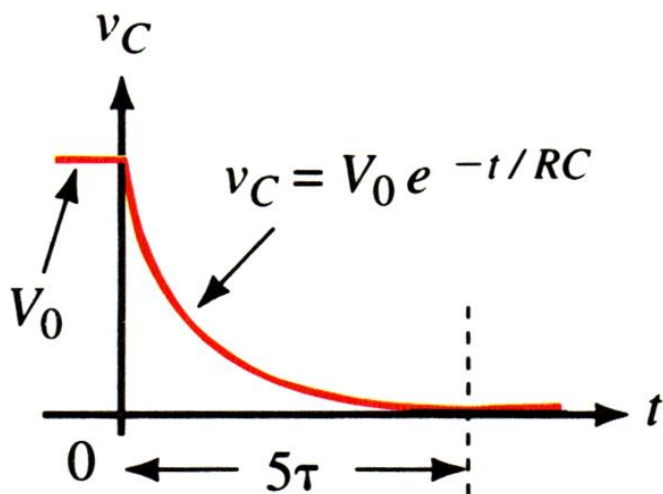- If a capacitor is charged to voltage V_0 and then discharged, the equations become

$$v_C = V_0 e^{-t/\tau}$$

$$v_R = -V_0 e^{-t/\tau}$$

$$i_C = -\frac{V_0}{R} e^{-t/\tau}$$

簡易看法：上頁（☆式）之 E 以 0 代入即可得（視為電壓源被移除）

$$v_C = V_0 e^{-t/RC}$$

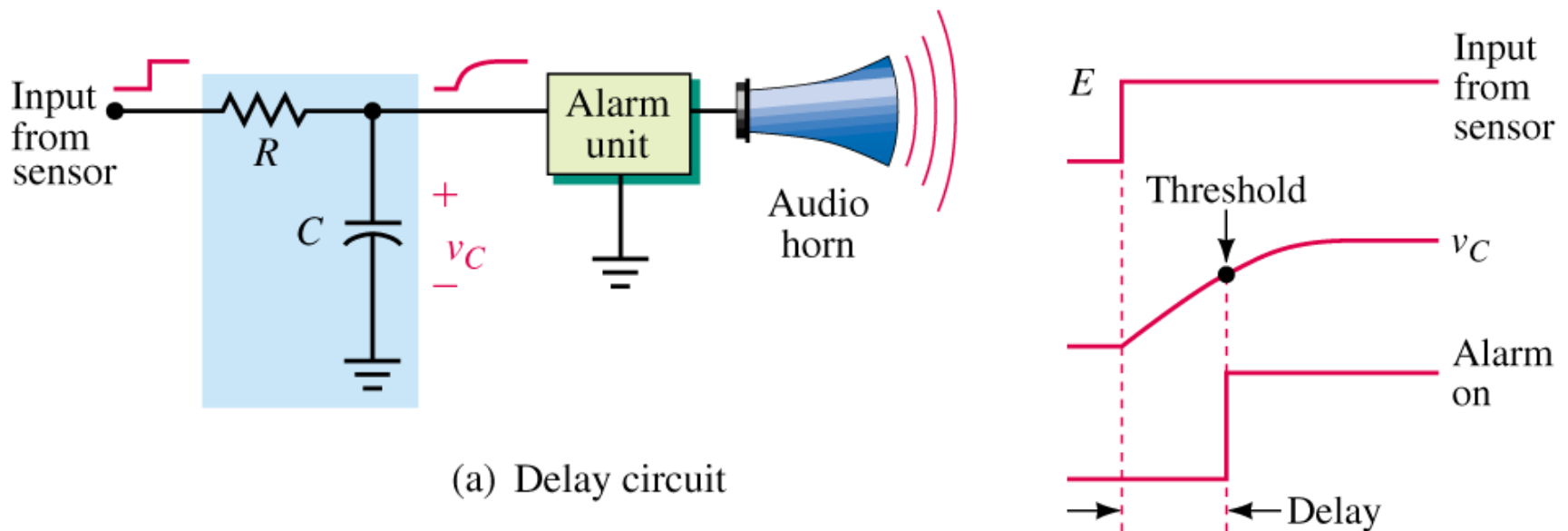$$i_C = -\frac{V_0}{R} e^{-t/RC}$$

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Well，瞭解RC電路充放電行為，so what? 然後呢？

延伸應用

- 相機內部的電容值選取，使得閃光燈能正確工作
  - 充電需多少秒，即足以驅動閃光燈
  - 瞭解釋放的瞬間電流大小，為閃光燈內嵌適當的保護電路
  - 估算消耗掉的能量(轉換成熱能散逸)，是否會讓機身過熱
- 低通(low-pass)濾波器 *http://bit.ly/1PW29X6*
- 電容式觸控技術 *http://bit.ly/1Qxk2IY*

An RC Timing Application

- RC circuits are used to create delays for alarm, motor control, and timing applications
- The alarm unit contains a threshold detector, and when the input to this detector exceeds a preset value, the alarm is turned on
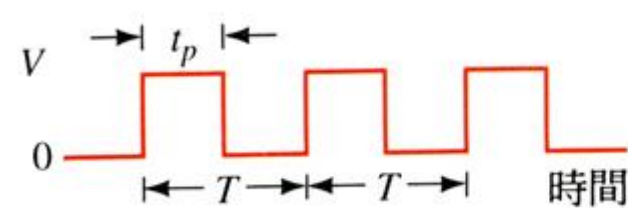
Input from sensor

R

C

$+$ v_C $-$

Alarm unit

Audio horn

(a) Delay circuit

E

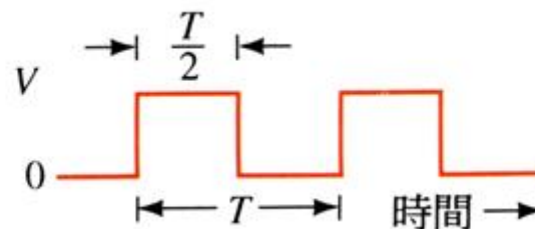Threshold

v_C

Input from sensor

Alarm on

Delay

(b)

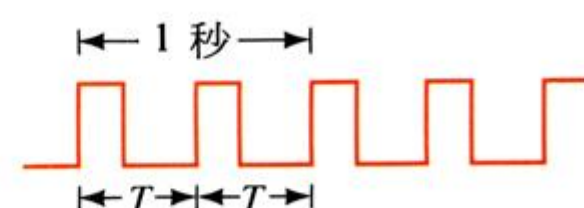# Pulse Response (脈波響應) of *RC* Circuits

- A pulse is a voltage or current that changes from one level to the other and back again

- A pulse train (脈波串列) is a repetitive stream of pulses

- Waveform with high time equal its low time is a square wave (方波)

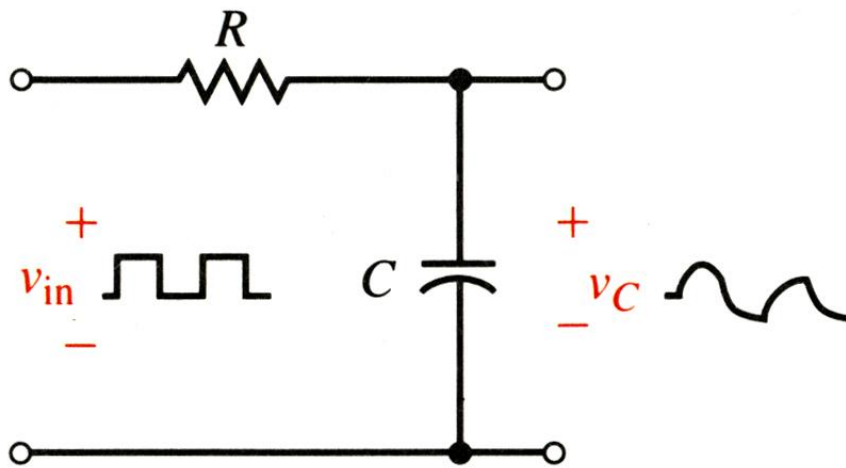- Length of each cycle of a pulse train is period T
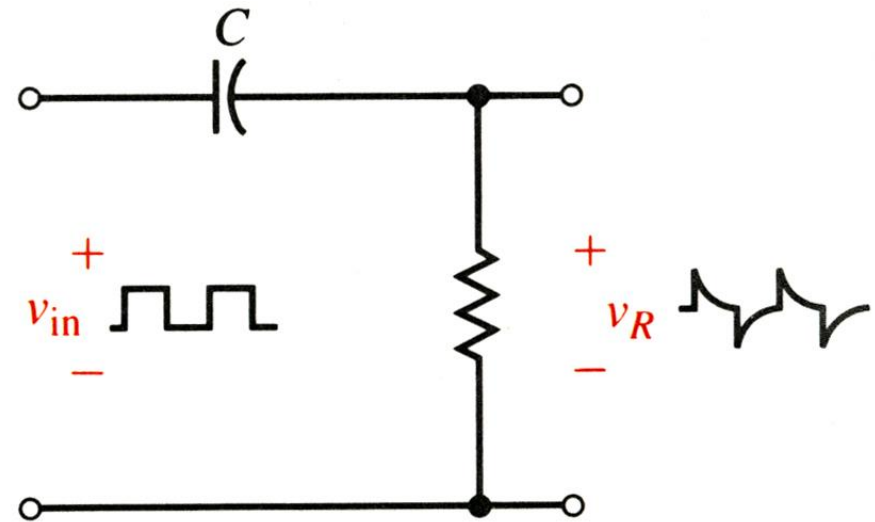

(a) 脈波串列，T為週期

(b) 方波

(c) Pulse repetition rate = 2 脈波/秒

Pulse Response of *RC* Circuits

- Pulse width relative to a circuit's time constant determines how it is affected by an RC circuit
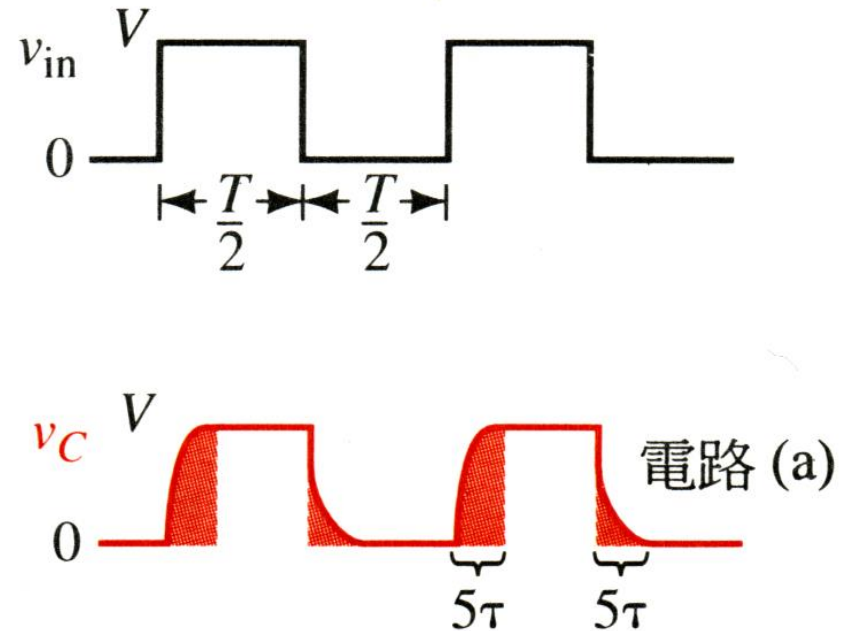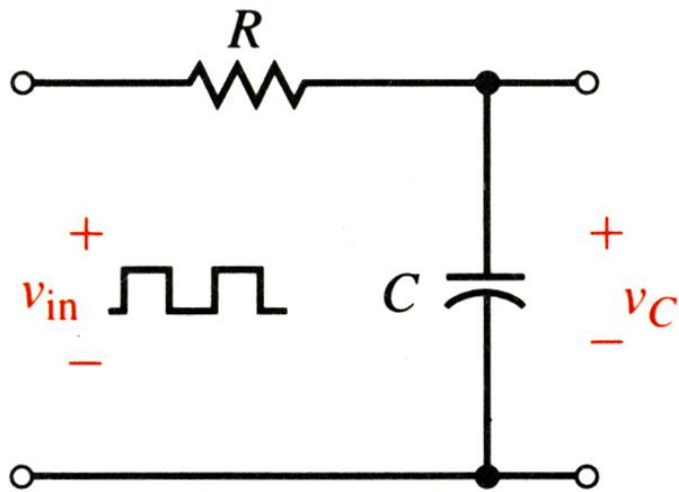

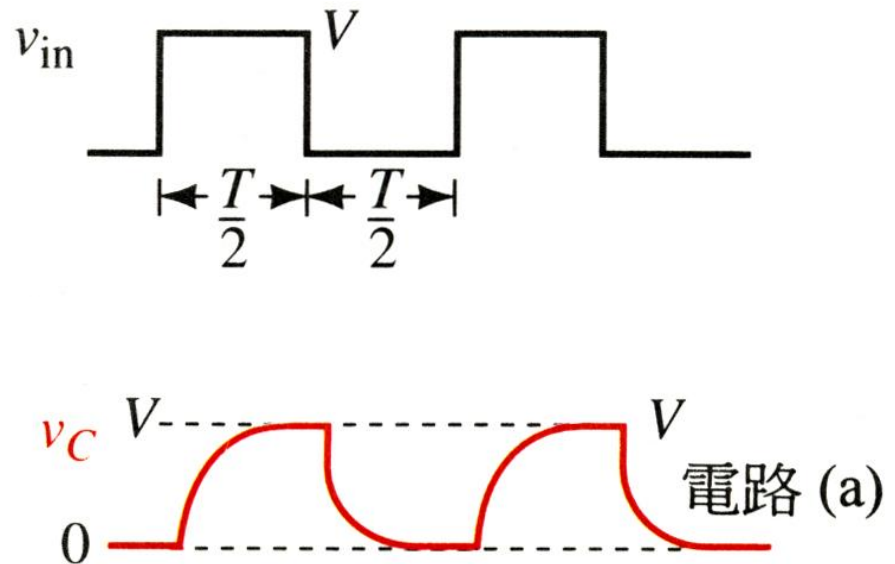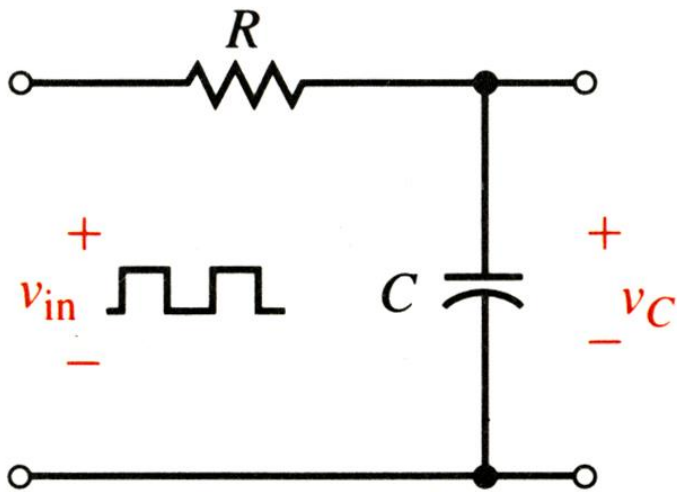(a) 跨接於 *C* 的輸出   (b) 跨接於 *R* 的輸出

Effect of Pulse Width (1/3)

- If pulse width >> 5τ, the capacitor charges and discharges fully
  - For circuit (a), charging and discharging occur at the transitions of the pulse

電路 (a)

  - 陰影區域代表電容充放電

Effect of Pulse Width (2/3)

- If pulse width = 5τ, the capacitor fully charges and discharges during each pulse

電路 (a)

Effect of Pulse Width (3/3)

- If pulse width $<< 5\tau$, the capacitor cannot fully charge and discharge
  - Switching occurs on the early part of the charging and discharging curves, $\therefore v_C$ is roughly triangular
  - $v_C \doteqdot$ integral of v_{in} ; v_C does not reach the steady state till 5 time constants

Effect of Pulse Width: 範例 (1/3)

- 設5V的方波輸入，脈寬為0.1秒且τ=0.1秒
  - 在第一個脈波(充電t=0.1s)末，v_c爬升到$5(1-e^{-0.1/0.1})$= $5(1-e^{-1})$=3.16V
    接著開始放電，直至第二個脈波的開端(歷時0.1s)，v_c從3.16V減少到$3.16e^{-0.1/0.1}$=$3.16e^{-1}$=1.16V
  - v_c從1.16V開始，0.1秒後爬升到3.59V，之後再從3.59V下降至$3.59e^{-1}$=1.32V
  - 繼續以上步驟，可算出後續變化值；RC電路花了5個常數時間才穩定

(a) 輸入波形

(b) 輸出電壓 υ_C

Effect of Pulse Width: 範例 (2/3)

```c
#include <stdio.h>
#include <stdlib.h>
#include <math.h>

#define MaxCycle 9   /* 考慮9個脈波 */

int main(int argc, char *argv[])
{
    int i;
    double E = 5, t = 0.1, V0 = 0, tau = 0.1;
    double VC_h[
    
    printf("5V的
    for (i = 0;
        V0 = VC_
        V0 = VC_
        printf("
        
    }
    
    system("paus
    return 0;
}
```
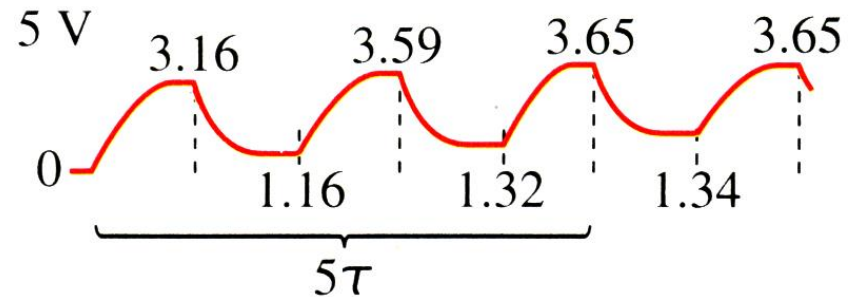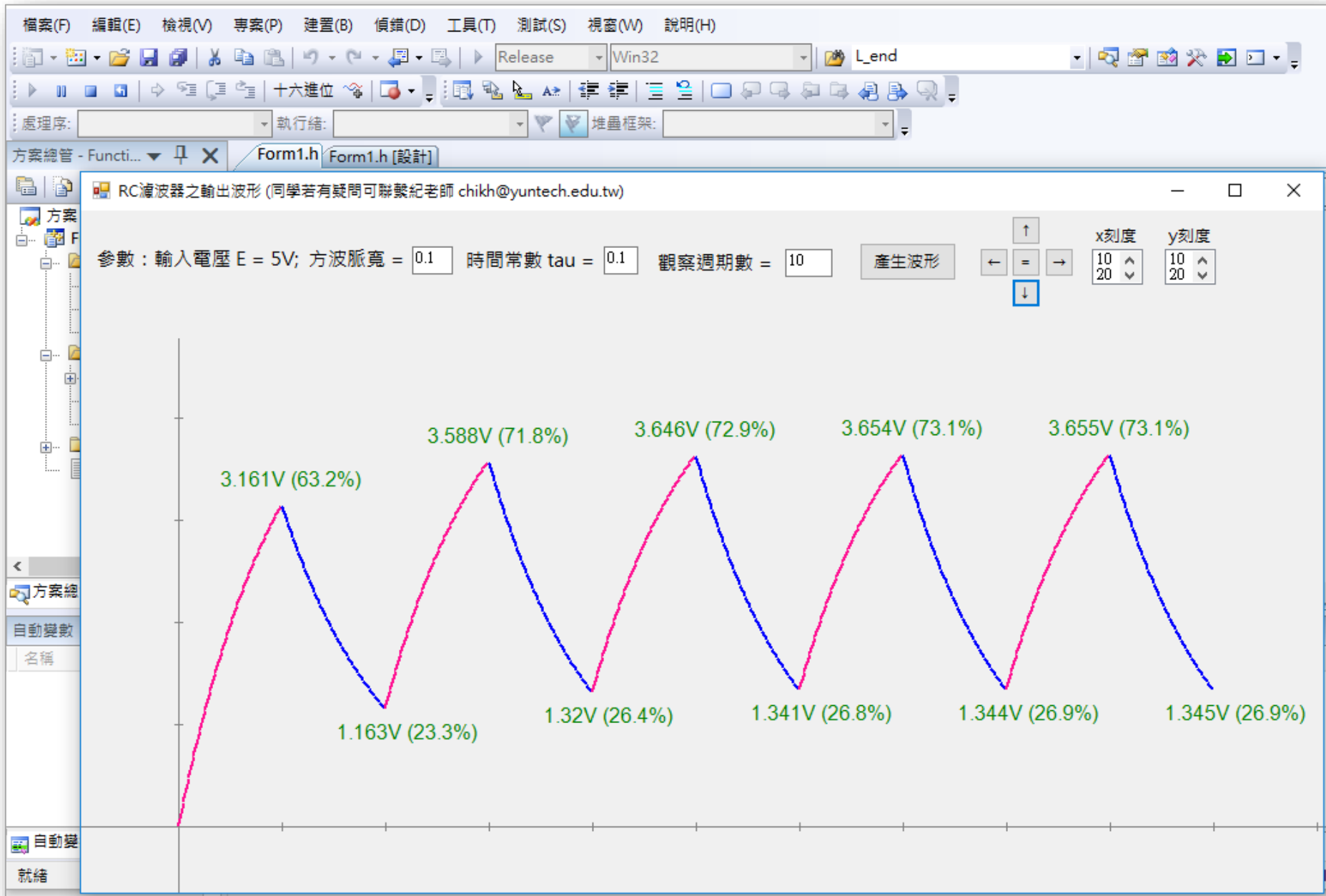
```
5V的方波輸入，脈寬0.1秒，時間常數tau=0.1秒
第1脈波：充電達3.1606V (63.21%)，放電至1.1627V (23.25%)
第2脈波：充電達3.5883V (71.77%)，放電至1.3201V (26.40%)
第3脈波：充電達3.6462V (72.92%)，放電至1.3414V (26.83%)
第4脈波：充電達3.6541V (73.08%)，放電至1.3443V (26.89%)
第5脈波：充電達3.6551V (73.10%)，放電至1.3446V (26.89%)
第6脈波：充電達3.6553V (73.11%)，放電至1.3447V (26.89%)
第7脈波：充電達3.6553V (73.11%)，放電至1.3447V (26.89%)
第8脈波：充電達3.6553V (73.11%)，放電至1.3447V (26.89%)
第9脈波：充電達3.6553V (73.11%)，放電至1.3447V (26.89%)
請按任意鍵繼續 . . .
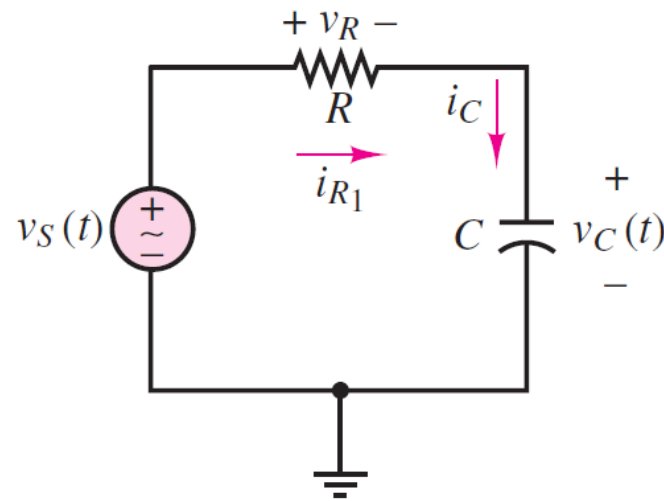```

# Effect of Pulse Width: 範例 (3/3)

# 輸入電源改為交流電之充放電方程式

若交流電源 $v_s(t) = V\cos\omega t$

方程式 $RC\dfrac{dv_C}{dt} + v_C = V\cos\omega t$

令 $\boxed{v_c(t) = A\sin\omega t + B\cos\omega t}$

$$A\omega\cos\omega t - B\omega\sin\omega t + \frac{1}{RC}(A\sin\omega t + B\cos\omega t) = \frac{1}{RC}V\cos\omega t$$

$$A = \frac{V\omega RC}{1 + \omega^2(RC)^2} \qquad B = \frac{V}{1 + \omega^2(RC)^2}$$

# 回顧學習目標

- 瞭解RC電路在生活中的應用
- 瞭解RC電路的工作原理
- 以方程式精確描述RC電路的充放電行為
  - 以微積分式表達電流與電壓之間的關係
  - 推導電路的特性
  - 延伸方程式的內涵，反思工程實務上的課題
- RC電路的暫態特性於其他工程的應用
- 以軟體工具探討RC電路的脈波響應
  - 程式撰寫
  - 不同的輸入形式與輸出的電壓之間的關係

# Remarks

- RC電路之充放電行為實為解<span style="color:blue">微分方程式</span>
  - 電路學、工程數學將再觸及
- 推導：輸入電源為交流電之RC電路充放電方程式
  - 練習：若$v_s(t)=V\sin\omega t$，則$v_C(t)$？
- 比較：RL(inductor)電路亦有類似的方程式形式


- 紀老師自行開發檢視RC電路脈波響應之程式碼
  - C版本：*https://bit.ly/3tJyv19*
  - Visual C++版本：*https://bit.ly/3A7oyx3*
  - 授權同學可自由使用(*chikh@yuntech.edu.tw*)